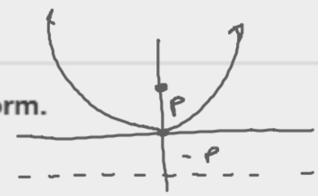
HOW TO

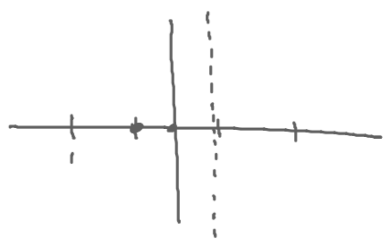


Given its focus and directrix, write the equation for a parabola in standard form.

1. Determine whether the axis of symmetry is the *x*- or *y*-axis.
   a. If the given coordinates of the focus have the form $(p, 0)$, then the axis of symmetry is the *x*-axis. Use the standard form $y^2 = 4px$.
   b. If the given coordinates of the focus have the form $(0, p)$, then the axis of symmetry is the *y*-axis. Use the standard form $x^2 = 4py$.
2. Multiply $4p$.
3. Substitute the value from Step 2 into the equation determined in Step 1.

Writing the Equation of a Parabola in Standard Form Given its Focus and Directrix

What is the equation for the parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $x = \frac{1}{2}$?



$p = -\frac{1}{2}$

$p = -\frac{1}{2}$

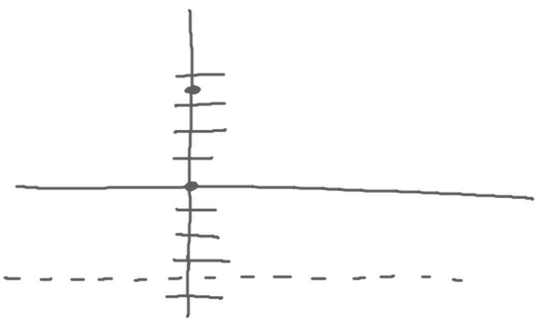
$4p = 4\left(-\frac{1}{2}\right)$

$= -2$

$y^2 = 4px$

$y^2 = -2x$

What is the equation for the parabola with focus $\left(0, \frac{7}{2}\right)$ and directrix $y = -\frac{7}{2}$?



$x^2 = 4py$

$p = \frac{7}{2}$

$4p = 4\left(\frac{7}{2}\right)$

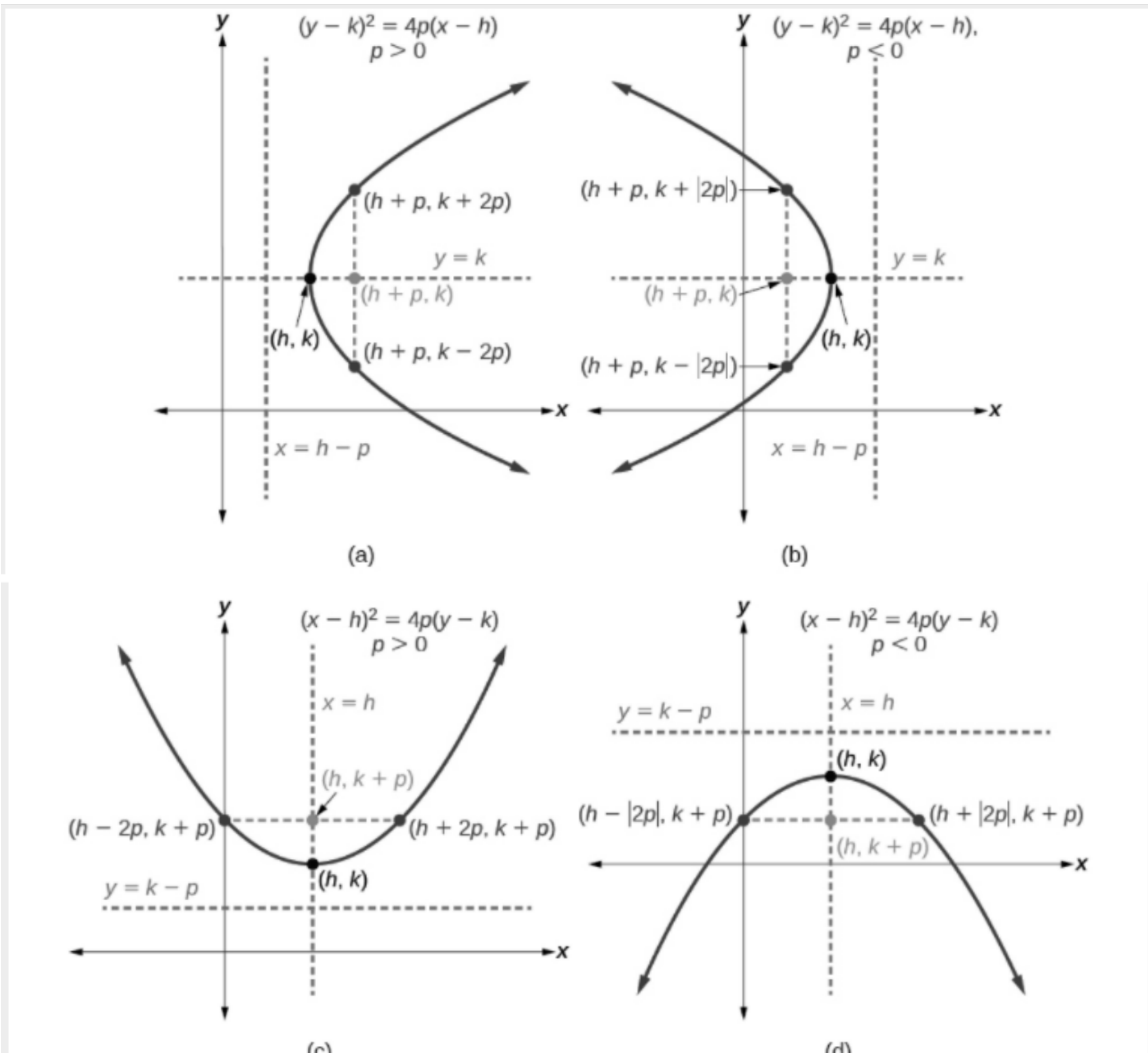
$4p = 14$

$x^2 = 14y$

STANDARD FORMS OF PARABOLAS WITH VERTEX (*H*, *K*)

Table 2 and Figure 9 summarize the standard features of parabolas with a vertex at a point (h, k).

Axis of Symmetry	Equation	Focus	Directrix	Endpoints of Latus Rectum
$y = k$	$(y-k)^2 = 4p(x-h)$	$(h+p, \quad k)$	$x = h - p$	$(h+p, \quad k \pm 2p)$
$x = h$	$(x-h)^2 = 4p(y-k)$	$(h, \quad k+p)$	$y = k - p$	$(h \pm 2p, \quad k+p)$


(y − k)² = 4p(x − h)
p > 0
(h + p, k + 2p)
y = k
(h + p, k)
(h, k)
(h + p, k − 2p)
x = h − p
(a)
(y − k)² = 4p(x − h),
p < 0
(h + p, k + |2p|)
y = k
(h + p, k)
(h, k)
(h + p, k − |2p|)
x = h − p
(b)
(x − h)² = 4p(y − k)
p > 0
x = h
(h, k + p)
(h − 2p, k + p)
(h + 2p, k + p)
(h, k)
y = k − p
(x − h)² = 4p(y − k)
p < 0
y = k − p
x = h
(h, k)
(h − |2p|, k + p)
(h + |2p|, k + p)
(h, k + p)
x
y

HOW TO

Given a standard form equation for a parabola centered at (*h*, *k*), sketch the graph.

1. Determine which of the standard forms applies to the given equation: $(y-k)^2=4p(x-h)$ or $(x-h)^2=4p(y-k)$.
2. Use the standard form identified in Step 1 to determine the vertex, axis of symmetry, focus, equation of the directrix, and endpoints of the latus rectum.

   a. If the equation is in the form $(y-k)^2=4p(x-h)$, then:

   - use the given equation to identify h and k for the vertex, (h,k)
   - use the value of k to determine the axis of symmetry, $y=k$
   - set $4p$ equal to the coefficient of $(x-h)$ in the given equation to solve for p. If $p>0$, the parabola opens right. If $p<0$, the parabola opens left.
   - use $h,k,$ and p to find the coordinates of the focus, $(h+p,\quad k)$
   - use h and p to find the equation of the directrix, $x=h-p$
   - use $h,k,$ and p to find the endpoints of the latus rectum, $(h+p,k\pm 2p)$

   b. If the equation is in the form $(x-h)^2=4p(y-k)$, then:

   - use the given equation to identify h and k for the vertex, (h,k)
   - use the value of h to determine the axis of symmetry, $x=h$
   - set $4p$ equal to the coefficient of $(y-k)$ in the given equation to solve for p. If $p>0$, the parabola opens up. If $p<0$, the parabola opens down.
   - use $h,k,$ and p to find the coordinates of the focus, $(h,\quad k+p)$
   - use k and p to find the equation of the directrix, $y=k-p$
   - use $h,k,$ and p to find the endpoints of the latus rectum, $(h\pm 2p,\quad k+p)$

3. Plot the vertex, axis of symmetry, focus, directrix, and latus rectum, and draw a smooth curve to form the parabola.

Graphing a Parabola with Vertex (*h*, *k*) and Axis of Symmetry Parallel to the *x*-axis

Graph $(y-1)^2 = -16(x+3)$. Identify and label the vertex, axis of symmetry, focus, directrix, and endpoints of the latus rectum.

$(y-k)^2 = 4p(x-h)$

Vertex $(-3,1)$

A.O.S. $y=1$

$4p=-16$

$p=-4$

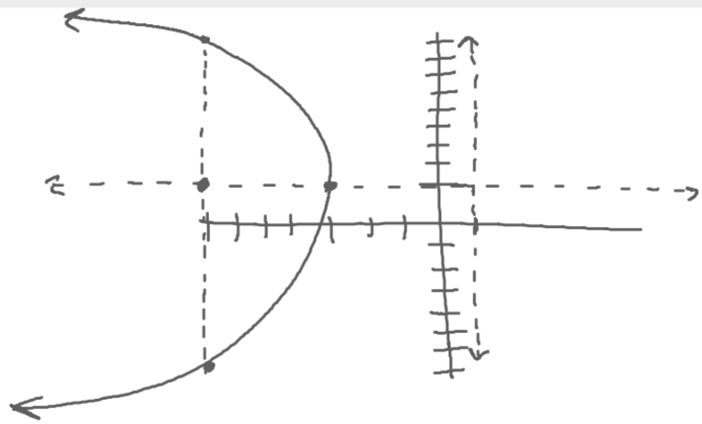
Focal $(-7,1)$

Directrix

$x=1$

Latus

$(-7,9)$ $(-7,-7)$



Graph $(y+1)^2 = 4(x-8)$. Identify and label the vertex, axis of symmetry, focus, directrix, and endpoints of the latus rectum.

Vertex $(8,-1)$

A.O.S. $y=-1$

Focus $(9,-1)$

Directrix

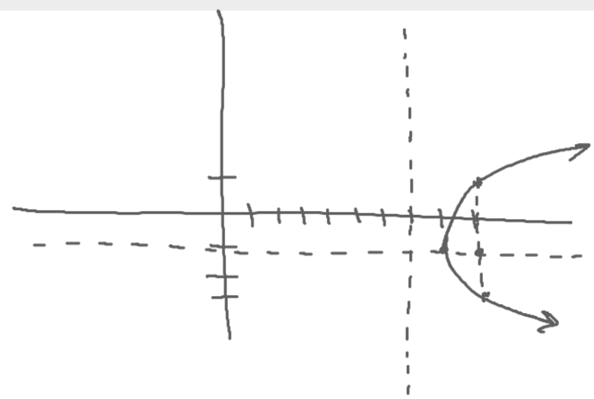
$x = 7$

Latus Endpoints

$(9,1)$ $(9,-3)$

$4p = 4$

$p = 1$



Graphing a Parabola from an Equation Given in General Form

$(x-h)^2 = 4p(y-k)$

Graph $x^2 - 8x - 28y - 208 = 0$. Identify and label the vertex, axis of symmetry, focus, directrix, and endpoints of the latus rectum.

$x^2 - 8x + 16 = 28y + 208 + 16$

$(x-4)^2 = 28y + 224$

$(x-4)^2 = 28(y+8)$

$4p = 28$

$p = 7$

V $(4,-8)$

A.O.S $x=4$

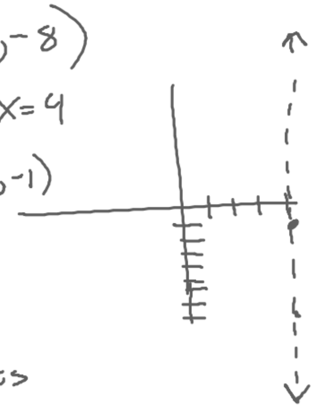
Focus $(4,-1)$

Directrix

$y=-15$

Latus Endpoints

$(-10,-1)$ $(18,-1)$



Graph $(x+2)^2 = -20(y-3)$. Identify and label the vertex, axis of symmetry, focus, directrix, and endpoints of the latus rectum.

V (-2, 3)

A.O.S x = -2

4p = -20

p = -5

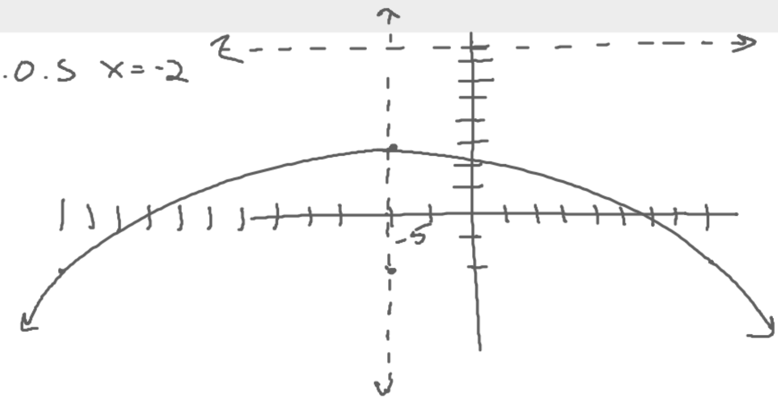
Focus (-2, -2)

Directrix

y = 8

Latus Endpoints

(-12, -2) (8, -2)



Solving Applied Problems Involving Parabolas

A cross-section of a design for a travel-sized solar fire starter is shown in Figure 13. The sun's rays reflect off the parabolic mirror toward an object attached to the igniter. Because the igniter is located at the focus of the parabola, the reflected rays cause the object to burn in just seconds.

- ⓐ Find the equation of the parabola that models the fire starter. Assume that the vertex of the parabolic mirror is the origin of the coordinate plane.
- ⓑ Use the equation found in part ⓐ to find the depth of the fire starter.

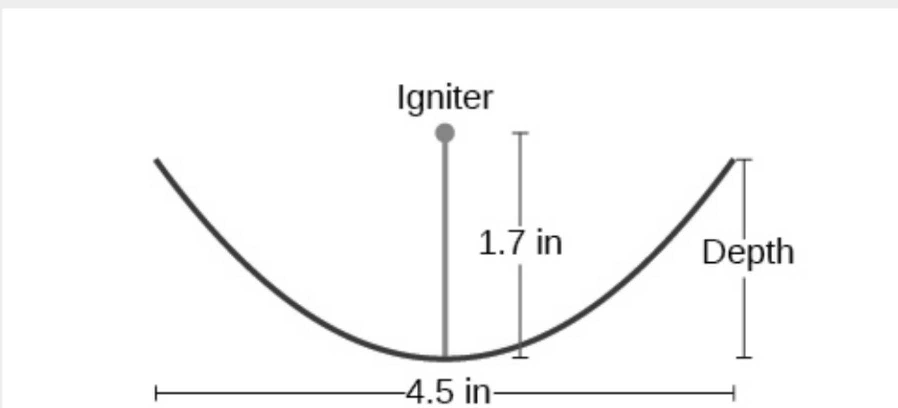


Figure 13 Cross-section of a travel-sized solar fire starter

Balcony-sized solar cookers have been designed for families living in India. The top of a dish has a diameter of 1600 mm. The sun’s rays reflect off the parabolic mirror toward the “cooker,” which is placed 320 mm from the base.

ⓐ Find an equation that models a cross-section of the solar cooker. Assume that the vertex of the parabolic mirror is the origin of the coordinate plane, and that the parabola opens to the right (i.e., has the x-axis as its axis of symmetry).

ⓑ Use the equation found in part ⓐ to find the depth of the cooker.